PHILOSOPHICAL TRANSACTIONS OF THE ROYAL SOCIETY A
MATHEMATICAL, PHYSICAL & ENGINEERING SCIENCES

Assessment of Defects: The C.E.G.B. Approach [and Discussion]

R. P. Harrison, I. Milne and T. G. F. Gray

Phil. Trans. R. Soc. Lond. A 1981 **299**, 145-153
doi: 10.1098/rsta.1981.0014

Phil. Trans. R. Soc. Lond. A **299**, 145–153 (1981) [145]
Printed in Great Britain

Assessment of defects: the C.E.G.B. approach

By R. P. Harrison† and I. Milne‡
Central Electricity Generating Board
† *Berkeley Nuclear Laboratories, Berkeley, Glos. GL13 9PB, U.K.*
‡ *Central Electricity Research Laboratories, Leatherhead, Surrey KT22 7SE, U.K.*

The failure of a structure is bounded by two extremes of behaviour: linear elastic and fully plastic. The procedures developed within the C.E.G.B. for assessing the effect of defects on the integrity of structures require the user to define the state of the structure in terms of each of these two extremes in turn. This is done by calculating the ratio of the applied loading condition to that required to cause failure. These two ratios then form the coordinates of a point on a failure assessment diagram. If the point falls inside the assessment line on that diagram, failure is avoided.

The approach is extremely versatile in operation. The methods used to define the two ratios are chosen and justified by the user. This allows full advantage to be taken of a wide range of such methods, and the successive refinement of those methods for selected aspects as the assessment proceeds and the critical areas are identified. In addition, methods for dealing with specific problems such as thermal and residual stresses and ductile crack growth are shown to be readily incorporated.

1. Introduction

When a cracked structure is loaded to failure it may follow any one of the paths shown schematically in figure 1.

When failure is by brittle mechanisms, even where there has been considerable plasticity before failure, it is abrupt; the load–displacement curve is discontinuous at failure. When failure is by ductile mechanisms it is progressive and the load–displacement curve is continuous. In such cases, failure can only occur if large displacements can be achieved in the locality of the crack tip. Which particular path in figure 1 is followed depends upon the geometry of the structure, the material properties at the location of the crack at the temperature of loading, the size and shape of the crack and the nature and magnitude of the load applied. It should be the function of any failure assessment procedure to choose the appropriate path to failure and define the critical conditions in relation to those applied.

The procedures developed within the Central Electricity Generating Board by Harrison *et al.* (1977) were designed to satisfy this function. When the procedures are followed to completion, all necessary plasticity corrections are automatically performed so that the critical conditions are properly identified. Judgement of the degree of safety of the structure is not made by recourse to rigid 'safety factors'. Instead, the user is encouraged to choose such factors objectively.

2. General philosophy

The basis of the C.E.G.B. procedures is the failure assessment diagram, figure 2, on which are plotted two parameters, S_r and K_r, evaluated under the appropriate loading conditions. The parameter S_r is a measure of how close the structure is to failure by plastic collapse and may be defined as

$$S_r = \sigma/\sigma_1(a),$$

where σ is the applied stress and $\sigma_1(a)$ is the plastic collapse stress as a function of the crack size. The parameter K_r is a measure of how close the structure is to linear elastic failure and may be defined as

$$K_r = K_I(a)/K_{Ic},$$

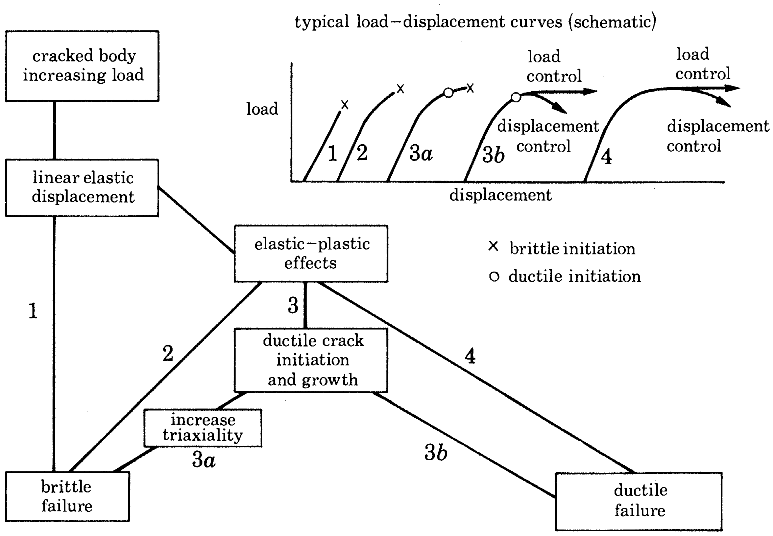

Figure 1. The five loading paths to failure.

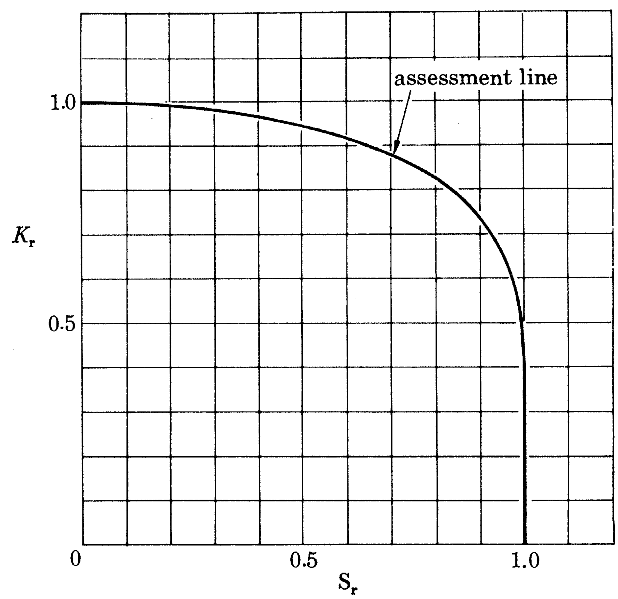

Figure 2. The failure assessment diagram.

PHILOSOPHICAL TRANSACTIONS OF THE ROYAL SOCIETY A MATHEMATICAL, PHYSICAL & ENGINEERING SCIENCES

where $K_I(a)$ is the linear elastic stress intensity factor and K_{Ic} is the fracture toughness at the initiation of crack growth.

The failure assessment line in figure 2 uses an elastic–plastic strip-yielding model (Dugdale 1960; Bilby *et al.* 1963; Heald *et al.* 1972) to interpolate between the two limits of behaviour (Dowling & Townley 1975). Failure is conceded if the assessment point, with coordinates (S_r, K_r), falls on or outside the assessment line.

The validity of this approach depends upon the failure assessment line being a reasonable approximation to the failure curve for any structural geometry. This has been demonstrated theoretically by Chell (1979). In addition, confirmation data have also been produced from laboratory specimens and structural tests (Harrison *et al.* 1979). The conclusions from the latter work are that not only is the failure assessment line a reasonable representation of a failure curve for structural geometries, but that when lower bound input data are used, it may be regarded as a failure avoidance line (figure 3).

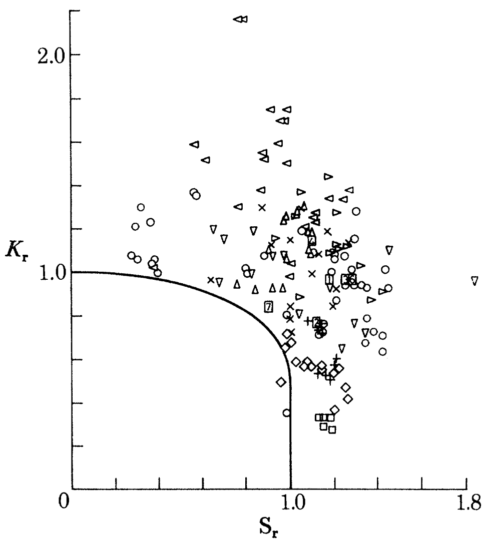

FIGURE 3. Validation of the failure assessment diagram.

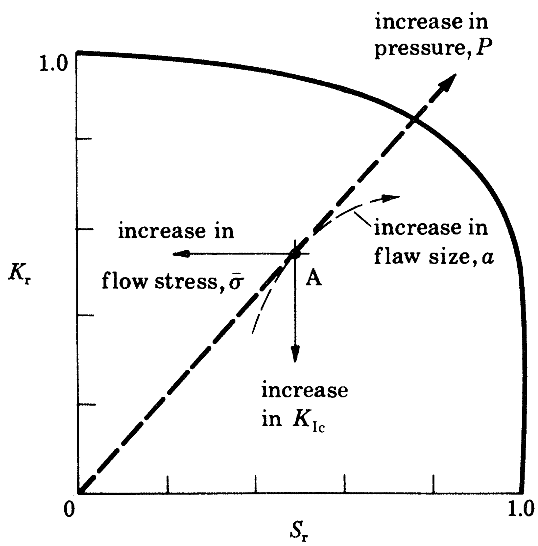

FIGURE 4. Sensitivity of the assessment point to input data.

The calculation of the two parameters S_r and K_r is all that is required to perform a complete analysis. Each of the parameters is treated independently. They have a simple functional dependence on such input data as crack depth, applied load or stress, and material properties. This makes it easy to define the locus of assessment points as one of the input parameters is varied while others are kept constant (figure 4). This feature facilitates sensitivity analyses in which the weaknesses in any assessment can be exposed and treated accordingly (Milne *et al.* 1980) while overall confidence is improved. By using this technique it is relatively easy for a user to judge the margins of safety objectively, taking the appropriate account of any uncertainties. The margins are not preordained and inflexibly hidden in some design curve.

3. THERMAL AND RESIDUAL STRESSES

Effects due to plasticity are normally incorporated into the procedure via the parameter S_r. Thermal and residual stresses, however, do not contribute to S_r but do induce plasticity.

The method to be adopted for dealing with these stresses (Harrison *et al.* 1980) has been described by Milne (1979*a*). It requires the separation of two types of stress, σ^p stresses, which contribute to plastic collapse, and σ^s, which do not. The parameter S_r is defined as in §1 but only uses σ^p stresses. The parameter K_r is defined as

$$K_r = K_r^p + K_r^s,$$

where K_r^p is defined as in §1, K_r^s contains all necessary plasticity corrections resulting from σ^s stresses, including the effect of σ^p stresses on these corrections. The corrections are based on first-order plasticity correction factors with an added degree of pessimism. Formby (1979) has provided validation evidence from some tests on centre-cracked plates that were loaded to failure by a combination of a well defined residual stress and an applied mechanical load.

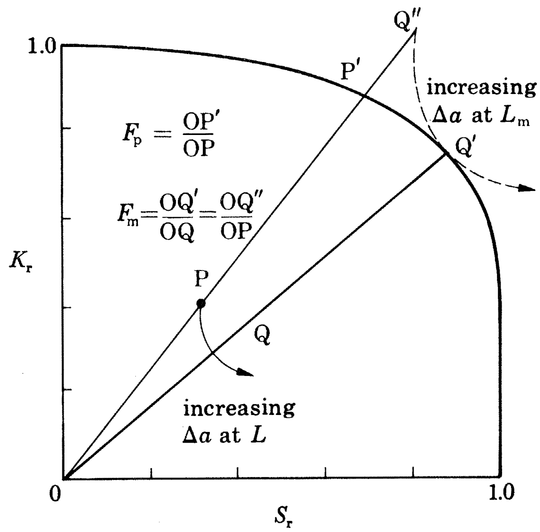

FIGURE 5. Principles of ductile crack growth analysis.

4. Ductile crack growth

So far in this paper, discussion has been limited to defining conditions for avoiding the initiation of cracking. This is implicit in the definition of K_{Ic} as a criterion for the initiation of cracking. However, when cracks grow by ductile mechanisms the load bearing capacity of a structure is often much higher than the load required to initiate the crack (Fearnehough *et al.* 1971). Recent advances in understanding the mechanics and mechanisms of ductile crack growth (Hutchinson & Paris 1979; Milne 1979*b*; Shih 1979; Turner 1979) extend the concept of the J integral into the ductile crack growth régime, and now allow estimation of the margin between the initiation load and the full load tolerance of the structure.

Chell (1979) has shown that the failure assessment curve may, for all practical purposes, be considered as a normalized J integral curve. Milne (1979*b*) was therefore able to extend the C.E.G.B. procedures to cover ductile crack growth, characterized by the J resistance curve, using the same simplified approach.

The parameters S_r and K_r are now generalized to include growth, so that

$$S_r = \frac{\sigma}{\sigma_1\{(a+\Delta a)/t\}}$$

and
$$K_r = \frac{K_I\{\sigma(a+\Delta a)/t\}}{K_\Omega(\Delta a)},$$
where Δa is the increment of crack growth associated with the resistance toughness $K_\Omega(\Delta a)$. This latter parameter is obtained from the elastic–plastic K or J resistance curve for the appropriate material.

A locus of assessment points at the applied load, L, is now constructed on the failure assessment line by keeping everything but Δa and $K_\Omega(\Delta a)$ constant (figure 5). The load tolerance of this structure, L_m, is then defined at the point where the load factor F reaches a maximum, as is the amount of crack growth up to this point.

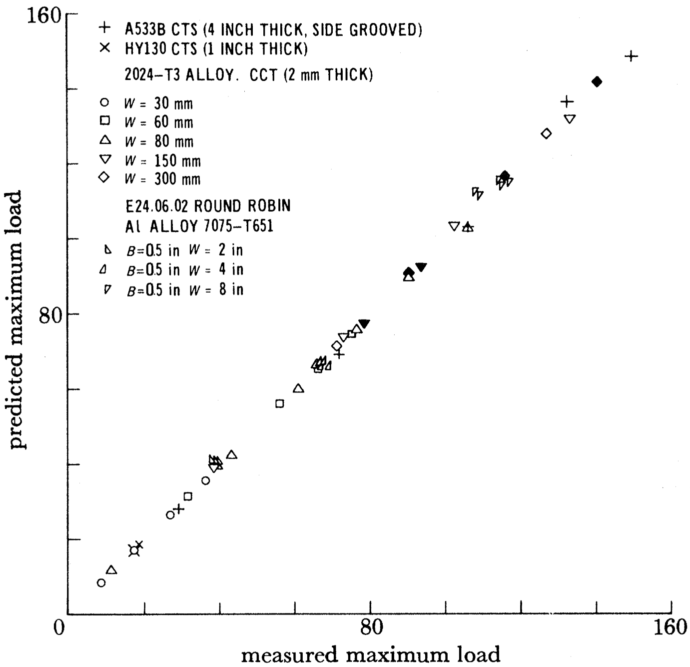

FIGURE 6. Validation of ductile crack growth analysis. Scale measurements: CT specimens in thousands of pounds-force (1 lbf ≈ 4.45 N); CCT specimens in kilonewtons (open symbols, scaled × 1; filled symbols, scale × $\frac{1}{2}$); E24.06.02 data in thousands of pounds-force (scale × 20).

The value for F can be derived graphically, for any assessment point, as indicated in figure 5, or by using the expression
$$F = (2/\pi S_r)\ \arccos\{\exp(-\tfrac{1}{8}\pi^2 S_r^2/K_r^2)\}.$$
The load capacity of the structure is then given by the product of this maximum value of F and the applied load, so that
$$L_m = F_{max} L.$$

A second locus of assessment points at this maximum load has been drawn on figure 5 that meets the failure assessment line at a tangent. This emphasizes the similarity between this approach and the resistance curve analyses procedures of other workers (Chell & Milne 1979; Hutchinson & Paris 1979; Shih 1979; Turner 1979).

Some evidence from experimental tests validating this approach has been provided by Bloom (1980). Additional data (K. H. Schwalbe & W. Setz, personal communication, 1979) have been added to this and plotted in figure 6.

PHILOSOPHICAL TRANSACTIONS OF THE ROYAL SOCIETY A MATHEMATICAL, PHYSICAL & ENGINEERING SCIENCES

The data are from tests on centre crack tension and compact tension geometries of specimens of 2024T3 and 70751–T651 Al alloys and A533B and HY130 steels in thicknesses ranging from 2 to 100 mm. Crack lengths varied from 0.1 to 0.9 of the specimen width. These results show that the measured and predicted values for maximum load were so close that a 45° line on figure 6 would obscure most of the points. This figure adds further support in validation of the basic failure assessment line.

5. Conclusions

Of the failure paths described in figure 1, the C.E.G.B. procedures are now capable of automatically choosing and following paths 1, 2, 3, 3*b* and 4, depending on the nature and extent of the input data available. The one remaining loading path, 3*a*, is sensitive to local conditions of stress and strain in the region of the advancing crack tip (Milne & Chell 1979) as well as local material properties. This problem is, of course, only experienced in or near the ductile brittle transition range, and does not occur in structures which are loaded at temperatures beyond this range. Where path 3*a* is a possibility, the analyses can still be performed, of course, up to the point of initiation of cracking, i.e. path 3.

Finally, the procedures described have been deliberately designed to confidently avoid failure rather than predict it. The analysis is greatly simplified because the necessary elastic and plastic calculations are independent of each other. This makes it easy to assess the sensitivity of the analyses to the varying confidence levels in the input parameters. It also facilitates the use of different methods for each of the calculations, as appropriate.

The invaluable contributions by Mr K. Loosemore and Dr A. R. Dowling of the Generation Development and Construction Division of the C.E.G.B. in developing the assessment procedure described are gratefully acknowledged.

This paper is published by permission of the Central Electricity Generating Board.

References (Harrison & Milne)

Bilby, B. A., Cottrell, A. H. & Swinden, K. H. 1963 *Proc. R. Soc. Lond.* A **272**, 304.
Bloom, J. M. 1980 *Int. J. Press. Vessels Piping* **8**, 215.
Chell, G. G. 1979 In *Elastic–plastic fracture* (*A.S.T.M. S.T.P.* no. 668), p. 581.
Chell, G. G. & Milne, I. 1979 *CEGB Report* no. RD/L/N87/79.
Dowling, A. R. & Townley, C. H. A. 1975 *Int. J. Press. Vessels Piping* **3**, 77.
Dugdale, D. S. 1960 *J. Mech. Phys. Solids* **8**, 100.
Fearnehough, G. D., Lees, G. M., Lowes, J. M. & Weiner, R. T. 1971 *Practical applications of fracture mechanics to pressure vessel technology*. London: Institution of Mechanical Engineers.
Formby, C. L. 1979 *CEGB Report* no. NW/SSD/RN116/79.
Harrison, R. P., Loosemore, K. & Milne, I. 1977 *CEGB Report* no. R/H/R6-Rev. 1.
Harrison, R. P., Loosemore, K. & Milne, I. 1979 *CEGB Report* no. R/H/R6-Supplement 1.
Harrison, R. P., Loosemore, K., Milne, I. & Dowling, A. R. 1980 *CEGB Report* no. R/H/R6-Rev. 2.
Heald, P. T., Spink, G. M. & Worthington, P. J. 1972 *Mater. Sci. Engng* **10**, 129.
Hutchinson, J. W. & Paris, P. C. 1979 In *Elastic–plastic fracture* (*A.S.T.M. S.T.P.* no. 668), p. 37.
Milne, I. 1979*a* Presented at Fifth Int. Conf. on Struct. Mechanics in Reactor Technol., Paper G1/3.
Milne, I. 1979*b* *Mater. Sci. Engng* **39**, 65.
Milne, I. & Chell, G. G. 1979 In *Elastic–plastic fracture* (*A.S.T.M. S.T.P.* no. 668), p. 358.
Milne, I., Harrison, R. P. & Dowling, A. R. 1980 In *Fourth Int. Conf. on Pressure Vessel Technol.*, paper C11/80. London: Institution of Mechanical Engineers.
Shih, C. F., de Lorenzi, H. G. & Andrews, W. R. 1979 In *Elastic–plastic fracture* (*A.S.T.M. S.T.P.* no. 668), p. 65.
Turner, C. E. 1979 In *Post-yield fracture mechanics* (ed. D. G. H. Latzko), p. 23. Applied Science Publ.

Discussion

T. G. F. Gray (*Department of Mechanics of Materials, University of Strathclyde, Glasgow, U.K.*). The C.E.G.B. fracture assessment method is very helpful in laying out many of the significant parameters in a given fracture problem in a clear way. However, on first acquaintance I was surprised that the semi-empirical relation used to interpolate between linear elasticity and limit load should turn out to be geometry independent. Admittedly, the C.E.G.B. equation is non-dimensionalized by the *linear-elastic* width correction factor but there is usually a strong interaction between finite width effects and plasticity, which is not explicitly accounted for in the modified B.C.S. model that the C.E.G.B. authors use.

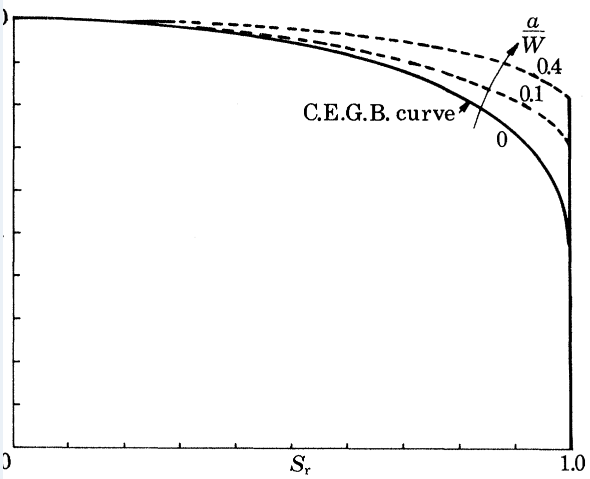

Figure 7. Double edge-notch.

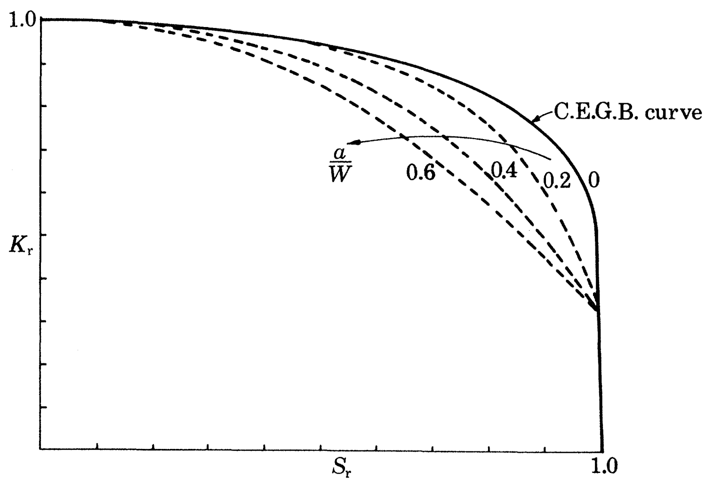

Figure 8. Single edge-notch.

It therefore seemed to be worth checking the form of the C.E.G.B. interpolation curve against any known relevant information. The finite-element strip-yield evaluations given in Hayes & Williams (1972) and Fenner (1974) for various finite-width geometries are of interest in this context, for although the yielding topography expected in the strip-yield model may differ somewhat from real cases (especially in plane strain) the pattern of yielding in the model should at least serve to *simulate* the distribution of yield level forces in a way that allows a fairly accurate determination of c.t.o.d. At any rate, there are better theoretical grounds for relying on these results than on the modified B.C.S. model, which can only be justified at the two extremes of linear elastic and limit load behaviour.

It should, however, be acknowledged, for the single edge-notch in tension, that the formulations of Hayes & Williams and Fenner do not include a compressive yield zone on the edge opposite the notch, as should be required for deep notches and/or high loads.

A comparison between the B.C.S. interpolation and the data of Hayes & Williams and Fenner is shown in figures 7 and 8. For this comparison, a semi-empirical fit to the large volume of data of Hayes & Williams and Fenner (see Gray (1977)) was transformed to the coordinate system used by the C.E.G.B. authors and plotted. For symmetrical edge-notches, the C.E.G.B. curve is seen to be conservative, and this conclusion also applies to the central internal crack and to the circumferential edge-crack cases, which I also examined but which are not shown. However, the single edge-crack case is clearly not conservative.

PHILOSOPHICAL TRANSACTIONS OF THE ROYAL SOCIETY A MATHEMATICAL, PHYSICAL & ENGINEERING SCIENCES

Parts of the dashed curves shown in figures 7 and 8 are of course extrapolations of the finite-element data, and some further comment is required, especially for the single edge-notch. First, even if attention is restricted to the region where the data are merely interpolated, the general implications of figure 8 would not be altered; secondly, these extrapolations match the correct limit loads for the geometry (which is all that has been claimed for the modified B.C.S. model); and, lastly, some tests that I carried out on single edge-notch specimens in the range $0.21 < a/W < 0.55$ show that the semi-empirical fit gives an entirely accurate prediction of c.t.o.d. despite any differences that may exist between the model and the real situation in terms of the distribution and growth of plasticity.

The purpose in bringing forward this discussion is not to throw cold water on the C.E.G.B. philosophy, which remains valid and useful; rather it is to encourage the authors to define any limitations on their method clearly, and if necessary to consider replacing the single interpolation curve by a family of curves or a 'zone of uncertainty'.

References

Fenner, D. N. 1974 *Int. J. Fract. Mech.* **10**, 71.
Gray, T. G. F. 1977 In *Fracture mechanics in engineering practice* (ed. P. Stanley), p. 123. London: Applied Science Publishers.
Hayes, D. J. & Williams, J. G. 1972 *Int. J. Fract. Mech.* **8**, 239.

R. P. Harrison and I. Milne. The use of a log sec equation to interpolate between the two limits does not require that all plasticity must follow a strip yielding model. Indeed, this would not be consistent with the plastic limit solutions that we recommend. Moreover, these plastic limit solutions explicitly include finite width correction factors.

The equation chosen for interpolation is not claimed to reproduce every elastic–plastic solution ever devised, but to facilitate an elastic–plastic analysis of a structure about which, in real terms, we often know very little. However, we do claim that the equation, as constrained by the two limits, follows broadly the lower bound to all reasonable elastic–plastic solutions yet generated, regardless of geometry. Minor differences between our solution and some solutions for specific geometries have been demonstrated by Chell (1979), but they are insignificant when compared with the uncertainties in K_{Ic}, flaw size and stress analysis normally encountered when assessing real problems.

It is our opinion that the solutions quoted by Dr Gray for single edge-notched tension geometries are not reasonable but are overpessimistic. Indeed, when rigorously applied, the strip yielding model is itself somewhat pessimistic for this geometry. Dr Gray concedes this in drawing attention to the inability of the model to cater for the compression zone at the back surface, an effect that would lift the curves and that increases with increasing crack size and amount of plasticity. This point is adequately demonstrated if the failure curves for these geometries are replotted with the use of more realistic finite element solutions which include the effects of the back surface compression zone (compare figure 2*b* of Chell (1979) with Dr Gray's figure). In addition our figure 3 includes results from single edge-notched specimens tested in tension.

We cannot comment on Dr Gray's experimental data, not having seen them, but have the following general comments to make on c.t.o.d. data:

(i) they cannot be measured directly on s.e.n.t. geometries;

(ii) indirect methods of measuring c.t.o.d., such as notch root contraction, are unreliable, monitoring as they do surface phenomena only;

(iii) the relation between c.t.o.d. and other fracture mechanics parameters, K or J, is far from clear and varies with geometry.

It is therefore our belief that a more reliable indication of the true state of the s.e.n.t. specimens would be obtained using K or J data, obtained from load–displacement curves.

Finally, we would claim that, within the limitations imposed by the uncertainties inherent in solving practical problems of engineering significance, the proposed procedure will enable a user to avoid failure.

MATHEMATICAL, PHYSICAL & ENGINEERING SCIENCES
THE ROYAL SOCIETY A
PHILOSOPHICAL TRANSACTIONS OF